**Topic**: Polar equation of an elliptical conic section

**Question**: Which equation represents the polar equation of the ellipse?

$$x^2 + 4m^2y^2 - 4m^2 = 0$$

**Answer choices**:

A $$r = \frac{4m}{\sqrt{\cos^2\theta + 2m^2\sin^2\theta}}$$

B $$r = \frac{4m}{\sqrt{\cos^2\theta + 4m\sin^2\theta}}$$

C $$r = \frac{2m}{\sqrt{\cos^2\theta + 4m^2\sin^2\theta}}$$

D $$r = \frac{2m}{\sqrt{\cos^2\theta + 4m\sin^2\theta}}$$

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**Solution**: C

For the equation we've been given

$$x^2 + 4m^2y^2 - 4m^2 = 0$$

we'll use the conversion equations

$$x = r\cos\theta$$

$$y = r\sin\theta$$

to substitute and get

$$(r\cos\theta)^2 + 4m^2\left(r\sin\theta\right)^2 - 4m^2 = 0$$

$$r^2\cos^2\theta + 4m^2r^2\sin^2\theta - 4m^2 = 0$$

$$r^2\left(\cos^2\theta + 4m^2\sin^2\theta\right) = 4m^2$$

$$r^2 = \frac{4m^2}{\cos^2\theta + 4m^2\sin^2\theta}$$

$$r = \sqrt{\frac{4m^2}{\cos^2\theta + 4m^2\sin^2\theta}}$$

$$r = \frac{2m}{\sqrt{\cos^2\theta + 4m^2\sin^2\theta}}$$

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**Topic**: Polar equation of an elliptical conic section

**Question**: The polar equation represents which of the following ellipses?

$$r = \frac{5\cos\theta - 2\sin\theta \pm \sqrt{68\sin^2\theta + 345\cos^2\theta - 10\sin(2\theta)}}{2\sin^2\theta + 10\cos^2\theta}$$

**Answer choices**:

A $5x^2 + y^2 + 10x - 2y - 16 = 0$

B $5x^2 + y^2 - 10x - 2y + 16 = 0$

C $5x^2 + y^2 - 10x + 4y + 16 = 0$

D $5x^2 + y^2 - 10x + 4y - 16 = 0$

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**Solution**: D

Choose

$$5x^2 + y^2 - 10x + 4y - 16 = 0$$

from answer choice D. Use the conversion equations

$$x = r\cos\theta$$

$$y = r\sin\theta$$

to substitute into the equation.

$$5(r\cos\theta)^2 + (r\sin\theta)^2 - 10(r\cos\theta) + 4(r\sin\theta) - 16 = 0$$

$$5r^2\cos^2\theta + r^2\sin^2\theta - 10r\cos\theta + 4r\sin\theta - 16 = 0$$

$$\left(5\cos^2\theta + \sin^2\theta\right)r^2 + 2\left(-5\cos\theta + 2\sin\theta\right)r - 16 = 0$$

Now this is a quadratic equation, and we can use the quadratic formula to find its roots. With $a = \left(5\cos^2\theta + \sin^2\theta\right)$, $b = 2\left(-5\cos\theta + 2\sin\theta\right)$, and $c = -16$, we plug into the quadratic formula and get

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$r = \frac{-\left(-5\cos\theta + 2\sin\theta\right) \pm \sqrt{\left(-5\cos\theta + 2\sin\theta\right)^2 - 4\left(5\cos^2\theta + \sin^2\theta\right)(-16)}}{2\left(5\cos^2\theta + \sin^2\theta\right)}$$

$$r = \frac{5\cos\theta - 2\sin\theta \pm \sqrt{25\cos^2\theta - 20\sin\theta\cos\theta + 4\sin^2\theta + 320\cos^2\theta + 64\sin^2\theta}}{10\cos^2\theta + 2\sin^2\theta}$$

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$$r = \frac{5\cos\theta - 2\sin\theta \pm \sqrt{-20\sin\theta\cos\theta + 345\cos^2\theta + 68\sin^2\theta}}{10\cos^2\theta + 2\sin^2\theta}$$

Using the trig identity $\sin(2x) = 2\sin x\cos x$, we get

$$r = \frac{5\cos\theta - 2\sin\theta \pm \sqrt{-10(2\sin\theta\cos\theta) + 345\cos^2\theta + 68\sin^2\theta}}{10\cos^2\theta + 2\sin^2\theta}$$

$$r = \frac{5\cos\theta - 2\sin\theta \pm \sqrt{-10(\sin(2\theta)) + 345\cos^2\theta + 68\sin^2\theta}}{10\cos^2\theta + 2\sin^2\theta}$$

$$r = \frac{5\cos\theta - 2\sin\theta \pm \sqrt{-10\sin(2\theta) + 345\cos^2\theta + 68\sin^2\theta}}{10\cos^2\theta + 2\sin^2\theta}$$

$$r = \frac{5\cos\theta - 2\sin\theta \pm \sqrt{68\sin^2\theta + 345\cos^2\theta - 10\sin(2\theta)}}{2\sin^2\theta + 10\cos^2\theta}$$

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**Topic**: Polar equation of an elliptical conic section

**Question**: Which two ellipses both have their centers at the origin?

**Answer choices**:

A $r^2 = \dfrac{m^4 + n^4}{m^6 \sin^2\theta_1 + n^6 \cos^2\theta_1}$ and $r^2 = \dfrac{m^6 - n^6}{m^6 \sin^2\theta_2 + n^6 \sin^2\theta_2 + 2}$

B $r^2 = \dfrac{m^4 n^4}{m^4 \sin^2\theta_1 + n^4 \cos^2\theta_1}$ and $r^2 = \dfrac{m^6 n^6}{m^6 \sin^2\theta_2 + n^6 \cos^2\theta_2}$

C $r^2 = \dfrac{m^4 n^4}{m^4 \sin^2\theta_1 + n^4 \cos^2\theta_1}$ and $r^2 = \dfrac{m^6 n^6}{m^6 \sin^2\theta_2 + n^6 \cos^2\theta_2 - 1}$

D $r^2 = \dfrac{m^4 n^4}{m^4 \sin^2\theta_1 + n^4 \cos^2\theta_1 - 1}$ and $r^2 = \dfrac{m^6 n^6}{m^6 \sin^2\theta_2 + n^6 \cos^2\theta_2}$

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**Solution**: B

Starting with answer choice B, we have the equations

$$r^2 = \frac{m^4 n^4}{m^4 \sin^2 \theta_1 + n^4 \cos^2 \theta_1}$$

$$r^2 = \frac{m^6 n^6}{m^6 \sin^2 \theta_2 + n^6 \cos^2 \theta_2}$$

We'll use the conversion equations $x = r\cos\theta$ and $y = r\sin\theta$, change them to

$$\cos\theta_1 = \frac{r}{x}$$

$$\sin\theta_1 = \frac{r}{y}$$

and then make substitutions into the first equation.

$$r^2 = \frac{m^4 n^4}{m^4 \sin^2 \theta_1 + n^4 \cos^2 \theta_1}$$

$$r^2 = \frac{m^4 n^4}{m^4 \left(\frac{y}{r}\right)^2 + n^4 \left(\frac{x}{r}\right)^2}$$

$$r^2 = \frac{m^4 n^4}{\frac{m^4 y^2 + n^4 x^2}{r^2}}$$

$$r^2 \left(\frac{m^4 y^2 + n^4 x^2}{r^2}\right) = m^4 n^4$$

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$$m^4y^2 + n^4x^2 = m^4n^4$$

$$\frac{m^4y^2}{m^4n^4} + \frac{m^4x^2}{m^4n^4} = \frac{m^4n^4}{m^4n^4}$$

$$\frac{y^2}{n^4} + \frac{x^2}{m^4} = 1$$

$$\frac{y^2}{(m^2)^2} + \frac{x^2}{(n^2)^2} = 1$$

This is the equation of an ellipse centered at the origin with $a = n^2$ and $b = m^2$.

Now substitute

$$\cos\theta_2 = \frac{r}{x}$$

$$\sin\theta_2 = \frac{r}{y}$$

into the second equation.

$$r^2 = \frac{m^6n^6}{m^6\sin^2\theta_2 + n^6\cos^2\theta_2}$$

$$r^2 = \frac{m^6n^6}{m^6\left(\frac{y}{r}\right)^2 + n^6\left(\frac{x}{r}\right)^2}$$

$$r^2 = \frac{m^6n^6}{m^6\left(\frac{y}{r}\right)^2 + n^6\left(\frac{x}{r}\right)^2}$$

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$$r^2 = \frac{m^4 n^4}{\frac{m^4 y^2 + n^4 x^2}{r^2}}$$

$$r^2 \left( \frac{m^6 y^2 + n^6 x^2}{r^2} \right) = m^6 n^6$$

$$m^6 y^2 + n^6 x^2 = m^6 n^6$$

$$\frac{m^6 y^2}{m^6 n^6} + \frac{m^6 x^2}{m^6 n^6} = \frac{m^6 n^6}{m^6 n^6}$$

$$\frac{y^2}{n^6} + \frac{x^2}{m^6} = 1$$

$$\frac{y^2}{\left(m^3\right)^2} + \frac{x^2}{\left(n^3\right)^2} = 1$$

This is the equation of an ellipse centered at the origin with $a = n^3$ and $b = m^3$.

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